

## Homework 3

Due October 28, 2015

Please submit your homework by email to haksun [dot] li {at} numericalmethod -dot- com.

Q1.

Show  $\max \sum_{j=1}^N u_j(1) \log(\pi_j)$  w.r.t each  $\pi_j$

$$\pi_j = \hat{u}_j(1) / \sum_{j=1}^N \hat{u}_j(1) = \hat{u}_j(1)$$

Hint:

Using a Lagrange multiplier subject to  $\sum_{j=1}^N \pi_j(1) = 1, \hat{u}_j(1) \geq 0$ .

Q2.

Show  $\max \sum_{t=2}^T \sum_{j=1}^N \sum_{k=1}^N v_{jk} \log a_{j,k}$  w.r.t. each  $v_{jk}$

$$v_{jk} = f_{jk} / \sum_{j=1}^N f_{jk}, \text{ where } f_{jk} = \sum_{t=2}^T \hat{v}_{jk}(t)$$

Q3.

For the M-step in the EM algorithm for HMM, write out the log-likelihood functions for the following distributions for their parameters. Note that some cannot be done analytically so a numerical maximization step is needed. For that step, specify the objective function to be maximized.

1. Beta
2. Binomial
3. Exponential
4. Gamma
5. LogNormal

Q4.

Prove the coefficients and the boundary conditions in lecture 3 notes.

Q5.

Prove the three adjustments in lecture 3 notes.