

NUMERICAL METHOD

Introduction to Algorithmic Trading Strategies
Lecture 1

Technical Analysis: A Scientific Perspective

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Speaker Profile



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Numerical Method Incorporation Limited

- ▶ A consulting firm in mathematical modeling, esp. algorithmic trading and quantitative investment.
- ▶ Products:
 - ▶ SuanShu: a modern math or numerical library
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- ▶ Customers:
 - ▶ brokerage houses and funds
 - ▶ multinational corporations
 - ▶ very high net worth individuals
 - ▶ gambling groups
 - ▶ academic institutions

References

- ▶ Emmanuel Acar, Stephen Satchell. Chapters 4, 5 & 6, Advanced Trading Rules, Second Edition. Butterworth-Heinemann; 2nd edition. June 19, 2002.
- ▶ Andrew W. Lo , Mark T. Mueller. Warning: Physics Envy May be Hazardous to Your Wealth! 2010
- ▶ Algorithmic Trading: Hidden Markov Models on Foreign Exchange Data. Patrik Idvall, Conny Jonsson. University essay from Linköpings universitet/Matematiska institutionen; Linköpings universitet/Matematiska institutionen. 2008.
- ▶ A tutorial on hidden Markov models and selected applications in speech recognition. Rabiner, L.R. Proceedings of the IEEE, vol 77 Issue 2, Feb 1989.

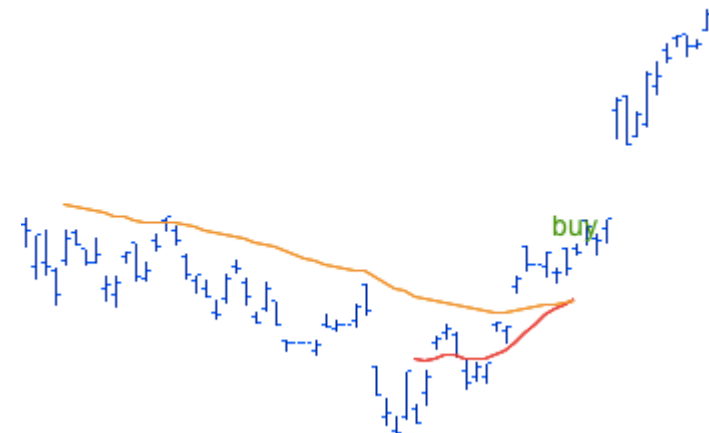
What is Quantitative Trading?

Quantitative Trading?

- ▶ Quantitative trading is the buying and selling of assets following the instructions computed from a set of proven **mathematical** models.
- ▶ The differentiation from other trading approaches or the emphasis is on **how** a strategy is generated and not on what strategy is created.
- ▶ It applies (rigorous) mathematics in all steps during trading strategy construction from the start to the end.

Moving Average Crossover as a TA

- ▶ A popular TA signal: Moving Average Crossover.
 - ▶ A crossover occurs when a faster moving average (i.e. a shorter period moving average) crosses above/below a slower moving average (i.e. a longer period moving average); then you buy/sell.
- ▶ In most TA book, it is then illustrated with an example of applying the strategy to a stock for a period of time to show the profits.



Technical Analysis is Not Quantitative Trading

- ▶ TA books merely describes the mechanics of strategies but seldom/never prove them.
- ▶ Appealing to common sense is not a mathematical proof.
- ▶ Conditional probabilities of outcomes are never computed. (Lo, Mamaysky, & Wang, 2000)
- ▶ Application of TA is more of an art (is it?) than a science.
 - ▶ How do you choose the parameters?

Fake Quantitative Models

- ▶ Data snooping
- ▶ Misuse of mathematics
- ▶ Assumptions cannot be quantified
- ▶ No model validation against the current regime
- ▶ Ad-hoc take profit and stop-loss
 - + why 2?
- ▶ How do you know when the model is invalidated?
- ▶ Cannot explain winning and losing trades
- ▶ Cannot be analyzed (systematically)

NM Quantitative Trading Research Process

1. Translate a vague trading intuition (hypothesis) into a concrete mathematical model.
2. Translate the mathematical symbols and equations into a computer program.
3. Strategy evaluation.
4. Live execution for making money.



Step 1 - Modeling

- ▶ Where does a trading idea come from?
 - ▶ Ex-colleagues
 - ▶ Hearsays
 - ▶ Newspapers, books
 - ▶ NOW TV, e.g., Moving Average Crossover (MA)
- ▶ A quantitative trading strategy is a math function, f , that at any given time, t , takes as inputs any information that the strategy cares and that is available, F_t , and gives as output the position to take, $f(t, F_t)$.

Step 2 - Coding

- ▶ The computer code enables analysis of the strategy.
 - ▶ Most study of a strategy cannot be done analytically.
 - ▶ We must resort to simulation.
- ▶ The same piece of code used for research and investigation should go straight into the production for live trading.
 - ▶ Eliminate the possibility of research-to-IT translation errors.

Step 3 – Evaluation/Justification

- ▶ Compute the properties of a trading strategy.
 - ▶ the P&L distribution
 - ▶ the holding time distribution
 - ▶ the stop-loss
 - ▶ the maximal drawdown
- ▶ <http://redmine.numericalmethod.com/projects/public/repository/svn-algoquant/show/core/src/main/java/com/numericalmethod/algoquant/execution/performance>

Step 4 - Trading

- ▶ Put in capitals incrementally.
- ▶ Install safety measures.
- ▶ Monitor the performance.
- ▶ Regime change detection.



The Basics

Quantitative Trading Strategy

- ▶ positions = $f(t, \mathcal{F}_t)$
- ▶ \mathcal{F}_t : filtration, the current information set.
 - ▶ E.g., $f(t, V_t, x_t)$,
 - ▶ t : current time
 - ▶ V_t : current portfolio value
 - ▶ x_t : current price

Probability Space

- ▶ A probability space is specified by $\{\Omega, \mathcal{F}, P\}$.
- ▶ Ω : the set of outcomes.
 - ▶ $\Omega = \{H, T\}$, as in a coin toss
- ▶ \mathcal{F} : the set of events. An event is a set of zero or more outcomes. An event is considered to have "happened" during an experiment when the outcome of the latter is an element of the event. Since the same outcome may be a member of many events, it is possible for many events to have happened given a single outcome.
 - ▶ $\mathcal{F} = \{\phi, H, T, \Omega\}$
- ▶ P : the probabilities of events, satisfying
 - ▶ $P(w) \geq 0$
 - ▶ $\sum P(w) = P(\Omega) = 1$
 - ▶ Often it is sufficient to specify the probabilities of outcomes to define the function.

Filtration (1)

- ▶ We flip a fair coin two times.

- ▶ Sample space, outcomes: $\Omega = \left\{ \begin{array}{l} \omega_1 = (H, H), \\ \omega_2 = (H, T), \\ \omega_3 = (T, H), \\ \omega_4 = (T, T) \end{array} \right\}$

Filtration (2)

- ▶ Σ -algebra, events or information available:

- ▶ $\mathcal{F}_0 = \{\phi, \Omega\}$

- ▶ $\mathcal{F}_1 = \left\{ \phi, \Omega, A = \left\{ \begin{array}{l} \omega_1 = (H, H), \\ \omega_2 = (H, T) \end{array} \right\}, \bar{A} = \left\{ \begin{array}{l} \omega_3 = (T, H), \\ \omega_4 = (T, T) \end{array} \right\} \right\}$

- ▶ Outcomes of the 2nd toss:

- ▶ $B = \left\{ \begin{array}{l} \omega_1 = (H, H), \\ \omega_3 = (T, H) \end{array} \right\}, \bar{B} = \left\{ \begin{array}{l} \omega_2 = (H, T), \\ \omega_4 = (T, T) \end{array} \right\}$

- ▶ $\mathcal{F}_2 = 2^\Omega$, σ -field generated by A, \bar{A}, B, \bar{B} .

- ▶ This set contains 16 events.

- ▶ $\mathcal{F}_0 \subset \mathcal{F}_1 \subset \mathcal{F}_2$ is the filtration generated by the coin flip process of two tosses.

Random Variable and Process

- ▶ Random variable, $X: \Omega \rightarrow E$
 - ▶ Assign a numeric value to each of the possible outcomes (not events).
- ▶ Random process: $\{X_t\}$
 - ▶ A collection of random variables indexed by time.

The Problem

- ▶ A short term security valuation problem.
- ▶ A regime identification problem to verify if the market is still in the regime assumed by our valuation. That is, are the model's assumptions valid in live trading.

Risk vs. Uncertainty - Easy

1. Complete Certainty: all past and future states of the system are determined exactly if initial conditions are fixed and known-nothing is uncertain.
 1. A coin with both sides being H.
2. Risk without Uncertainty: no statistical inference is needed, because we know the relevant probability distributions exactly, and while we do not know the outcome of any given wager, we know all the rules and the odds, and no other information relevant to the outcome is hidden.
 1. A fair coin.

Risk vs. Uncertainty - Challenging

- ▶ Fully Reducible Uncertainty: situations in which randomness can be rendered arbitrarily close to Level-2 uncertainty with sufficiently large amounts of data using the tools of statistical analysis.
 - ▶ An unfair coin.
- ▶ Partially Reducible Uncertainty: situations in which there is a limit to what we can deduce about the underlying phenomena the data.
 - ▶ Stochastic or time-varying parameters that vary too frequently to be estimated accurately.
 - ▶ A bag of many different unfair coins.
 - ▶ Complex non-linearities.
 - ▶ The dependencies on the unknowable.

Risk vs. Uncertainty – Divinity!

- ▶ Irreducible Uncertainty: this type of uncertainty is the domain of philosophers and religious leaders, who focus on not only the unknown, but [also] the unknowable.
 - ▶ Is there God who plays dice with the quantum coin, say, spin- $\frac{1}{2}$.
- ▶ Zen Uncertainty: attempts to understand uncertainty, are mere illusions; there is only suffering.

Information Ratio

- ▶ $IR = IC * \sqrt{\text{Breadth}}$
 - ▶ IR: information ratio
 - ▶ IC: information coefficient, e.g., correlation between forecasts and actuals
 - ▶ Breadth: number of independent bets
- ▶ For the same IR, we can be less accurate if we can trade more, e.g., intra-day trading.
 - ▶ Monthly vs. Daily
 - ▶ $IC_M * \sqrt{B_M} = IC_D * \sqrt{B_D}$
 - ▶ $IC_D = IC_M * \sqrt{B_M} / \sqrt{B_D} = IC_M / 5$
 - ▶ Daily vs. hourly
 - ▶ $IC_D * \sqrt{B_D} = IC_H * \sqrt{B_H}$
 - ▶ $IC_H = IC_D * \sqrt{B_D} / \sqrt{B_H} = IC_M / 2.45$
- ▶ Excellent IC: 0.05

Returns

- ▶ Real return = $s_1 = \frac{P_1 - P_0}{P_0} = \frac{P_1}{P_0} - 1$
- ▶ Log return = $r_1 = \log \frac{P_1}{P_0} = \log P_1 - \log P_0$
 - ▶ Time additive: the compound return for n period =
 $r_1 + \dots + r_n = \log \frac{P_n}{P_0}$
 - ▶ Normality: in quantitative finance, we often assume the prices are log-normally distributed to avoid the violation of limited liability.
 - ▶ Normality is preserved under addition, hence return compounding.
 - ▶ Total return is also normal.
 - ▶ Good approximation to real return: $r = \log(1 + s) \approx s$

The Quantitative Trading Research Process

Moving Average Crossover as a Quantitative Trading Strategy

- ▶ There are many mathematical justifications to Moving Average Crossover.
 - ▶ weighted Sum of lags of a time series
 - ▶ Kuo, 2002
- ▶ Whether a strategy is quantitative or not depends not on the strategy itself but
 - ▶ entirely on the process to construct it;
 - ▶ or, whether there is a scientific justification to prove it.

Step 2 - Modeling

- ▶ Two moving averages: slower (n) and faster (m).
- ▶ Monitor the crossovers.
- ▶ $B_t = \left(\frac{1}{m} \sum_{j=0}^{m-1} P_{t-j}\right) - \left(\frac{1}{n} \sum_{j=0}^{n-1} P_{t-j}\right), n > m$
- ▶ Long when $B_t \geq 0$.
- ▶ Short when $B_t < 0$.



How to Choose n and m ?

- ▶ It is an art, not a science (so far).
- ▶ They should be related to the length of market cycles.
- ▶ Different assets have different n and m .
- ▶ Popular choices:
 - ▶ (250, 5)
 - ▶ (250, 20)
 - ▶ (20, 5)
 - ▶ (20, 1)
 - ▶ (250, 1)



Two Simplifications

- ▶ Reduce the two dimensional problem to a one dimensional problem.
- ▶ Replace arithmetic averages with geometric averages.
 - ▶ This is so that we can work with log returns rather than prices.

AMA(n, 1)

- ▶ $B_t \geq 0$ iff $P_t \geq \left(\frac{1}{n} \sum_{j=0}^{n-1} P_{t-j}\right)$
- ▶ $B_t < 0$ iff $P_t < \left(\frac{1}{n} \sum_{j=0}^{n-1} P_{t-j}\right)$



GMA(n, 1)

- ▶ $B_t \geq 0$ iff $P_t \geq \left(\prod_{j=0}^{n-1} P_{t-j} \right)^{\frac{1}{n}}$
 - ▶ $R_t \geq - \sum_{j=1}^{n-2} \frac{n-(j+1)}{n-1} R_{t-j}$ (by taking log)
- ▶ $B_t < 0$ iff $P_t < \left(\prod_{j=0}^{n-1} P_{t-j} \right)^{\frac{1}{n}}$
 - ▶ $R_t < - \sum_{j=1}^{n-2} \frac{n-(j+1)}{n-1} R_{t-j}$ (by taking log)



What is n ?

- ▶ $n = 2$
- ▶ $n = \infty$



Acar Framework

- ▶ Acar (1993): to investigate the probability distribution of realized returns from a trading rule, we need
 - ▶ the explicit specification of the trading rule
 - ▶ the underlying stochastic process for asset returns
 - ▶ the particular return concept involved



Empirical Properties of Financial Time Series

- ▶ Asymmetry
- ▶ Fat tails

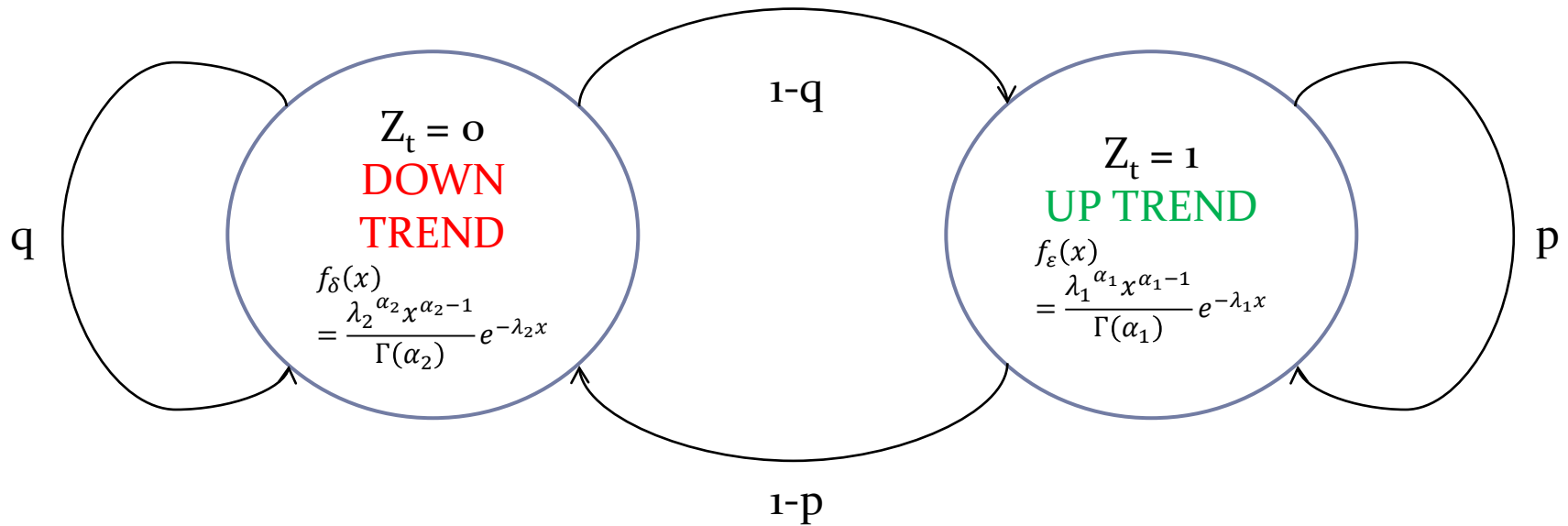


Knight-Satchell-Tran Intuition

- ▶ Stock returns staying going up (down) depends on
 - ▶ the realizations of positive (negative) shocks
 - ▶ the persistence of these shocks
- ▶ Shocks are modeled by gamma processes.
- ▶ Persistence is modeled by a Markov switching process.



Knight-Satchell-Tran Z_t

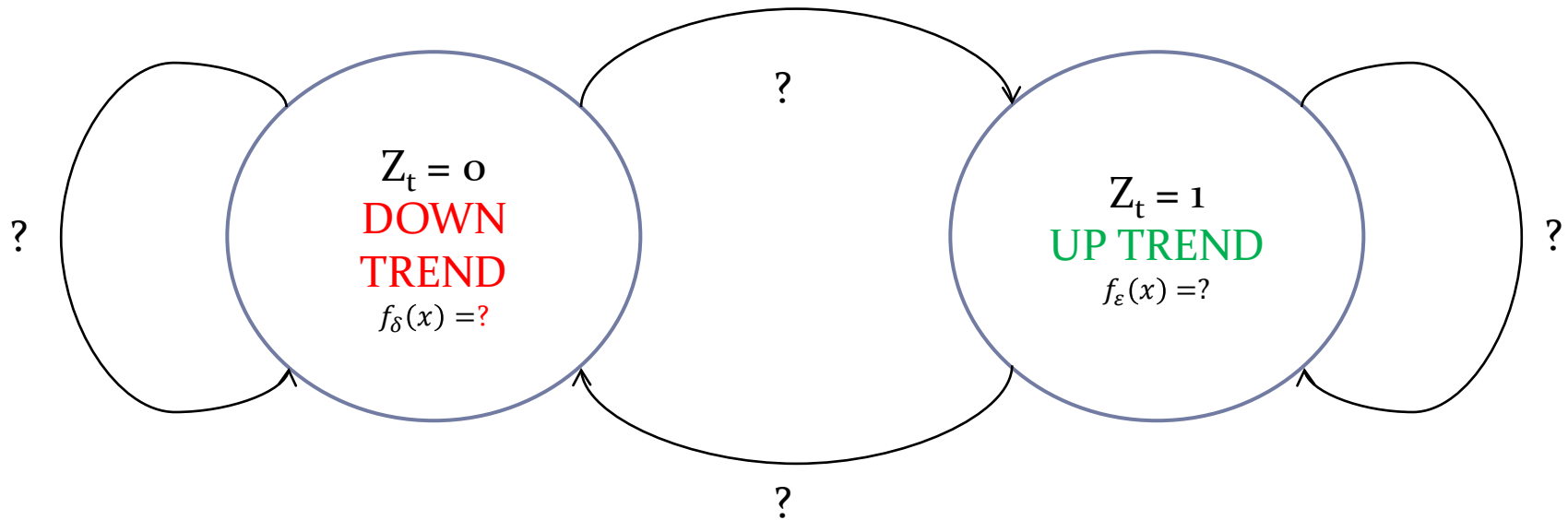


Knight-Satchell-Tran Process

- ▶ $R_t = \mu_l + Z_t \varepsilon_t - (1 - Z_t) \delta_t$
 - ▶ μ_l : long term mean of returns, e.g., 0
 - ▶ ε_t, δ_t : positive and negative shocks, non-negative, i.i.d
- ▶ $f_\varepsilon(x) = \frac{\lambda_1^{\alpha_1} x^{\alpha_1-1}}{\Gamma(\alpha_1)} e^{-\lambda_1 x}$
- ▶ $f_\delta(x) = \frac{\lambda_2^{\alpha_2} x^{\alpha_2-1}}{\Gamma(\alpha_2)} e^{-\lambda_2 x}$



Estimation of Parameters



Markov Property

- ▶ Given the current information available at time $(t - 1)$, the history, e.g., path, is irrelevant.
- ▶ $P(q_t | q_{t-1}, \dots, q_1) = P(q_t | q_{t-1})$
- ▶ Consistent with the weak form of the efficient market hypothesis.

Hidden Markov Chain

- ▶ Only observations are observable (duh).
- ▶ World states may not be known (hidden).
 - ▶ We want to model the hidden states as a Markov Chain.
- ▶ Two assumptions:
 - ▶ Markov property
 - ▶ $P(\omega_t | q_{t-1}, \dots, q_1, \omega_{t-1}, \dots, \omega_1) = P(\omega_t | q_t)$

Problems

▶ Likelihood

- ▶ Given the parameters, λ , and an observation sequence, Ω , compute $P(\Omega|\lambda)$.

▶ Decoding

- ▶ Given the parameters, λ , and an observation sequence, Ω , determine the best hidden sequence Q .

▶ Learning

- ▶ Given an observation sequence, Ω , and HMM structure, learn λ .

Learning as a Maximization Problem

- ▶ Our objective is to find λ that maximizes $P(\Omega|\lambda)$.
- ▶ For any given λ , we can compute $P(\Omega|\lambda)$.
- ▶ Then solve a maximization problem.
- ▶ Algorithms:
 - ▶ Nelder-Mead
 - ▶ Baum-Welch (E-M algorithm)

Stationary State

- ▶ $\Pi = \frac{1-q}{2-p-q}$
- ▶ $R_t = \mu_l + \varepsilon_t \geq \mu_l$, with probability Π
- ▶ $R_t = \mu_l - \delta_t < \mu_l$, with probability $1 - \Pi$



Step 3 – Evaluation/Justification

- ▶ Assume the long term mean is 0, $\mu_l = 0$.
- ▶ When $n = 2$,
 - ▶ $(B_t \geq 0) \equiv (R_t \geq 0) \equiv (Z_t = 1)$
 - ▶ $(B_t < 0) \equiv (R_t < 0) \equiv (Z_t = 0)$



GMA(2, 1) – Naïve MA Trading Rule

- ▶ Buy when the asset return in the present period is positive.
- ▶ Sell when the asset return in the present period is negative.



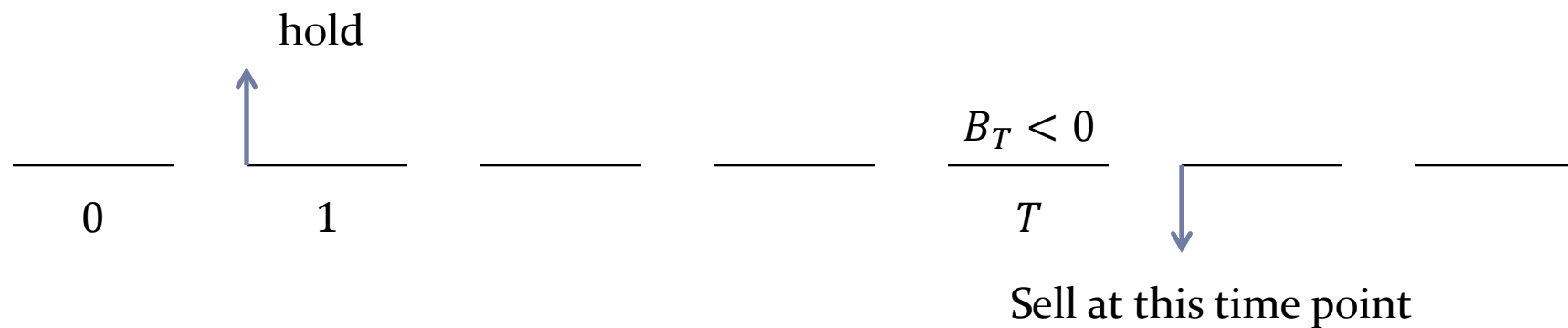
Naïve MA Conditions

- ▶ The expected value of the positive shocks to asset return \gg the expected value of negative shocks.
- ▶ The positive shocks persistency \gg that of negative shocks.



T Period Returns

▶ $RR_T = \sum_{t=1}^T R_t \times I_{\{B_{t-1} \geq 0\}}$



Holding Time Distribution

- ▶ $P(N = T)$
- ▶ $= P(B_T < 0, B_{T-1} \geq 0, \dots, B_1 \geq 0, B_0 \geq 0)$
- ▶ $= P(Z_T = 0, Z_{T-1} = 1, \dots, Z_1 = 1, Z_0 = 1)$
- ▶ $= P(Z_T = 0, Z_{T-1} = 1, \dots, Z_1 = 1 | Z_0 = 1)P(Z_0 = 1)$
- ▶ $= \begin{cases} \Pi p^{T-1} (1 - p), & T \geq 1 \\ 1 - \Pi, & T = 0 \end{cases}$



Conditional Returns Distribution (1)

- ▶ $\Phi_{RR_T|N=T}(s) = \mathbb{E} \left[e^{\{i[\sum_{t=1}^T R_t \times I_{\{B_{t-1} \geq 0\}}]s\}} | N = T \right]$
- ▶ $= \mathbb{E} \left[e^{\{i[\sum_{t=1}^T R_t \times I_{\{B_{t-1} \geq 0\}}]s\}} | B_T < 0, B_{T-1} \geq 0, \dots, B_0 \geq 0 \right]$
- ▶ $= \mathbb{E} \left[e^{\{i[\sum_{t=1}^T R_t]s\}} | Z_T = 0, Z_{T-1} = 1, \dots, Z_1 = 1 \right]$
- ▶ $= \mathbb{E} \left[e^{\{i[\varepsilon_1 + \dots + \varepsilon_{T-1} - \delta_T]s\}} \right]$
- ▶ $= \begin{cases} \Phi_\varepsilon^{T-1}(s) \Phi_\delta(-s), & T \geq 1 \\ \Phi_\delta(-s), & T = 0 \end{cases}$



Unconditional Returns Distribution (2)

▶ $\Phi_{RR_T}(s) =$

$$\sum_{T=0}^{\infty} \mathbb{E} \left[e^{i \left[\sum_{t=1}^T R_t \times I_{\{B_{t-1} \geq 0\}} \right] s} \mid N = T \right] P(N = T)$$

▶ $=$

$$\sum_{T=1}^{\infty} \Pi p^{T-1} (1-p) \Phi_{\varepsilon}^{T-1}(s) \Phi_{\delta}(-s) + (1-\Pi) \Phi_{\delta}(-s)$$

▶ $= (1-\Pi) \Phi_{\delta}(-s) + \Pi (1-p) \frac{\Phi_{\delta}(-s)}{1-p \Phi_{\varepsilon}(s)}$



Long-Only Returns Distribution

- ▶ $\Phi_{RR_T}(s|R_0 \geq 0) = \frac{(1-p)\Phi_\delta(-s)}{1-p\Phi_\varepsilon(s)}$
- ▶ Proof: make $P(Z_0 = 1) = \Pi = 1$



Expected Returns

- ▶ $E(RR_T) = -i\Phi_{RR_T}'(0)$
- ▶ $= \frac{1}{1-p} \{\Pi p \mu_\varepsilon - (1-p)\mu_\delta\}$
- ▶ When is the expected return positive?
 - ▶ $\mu_\varepsilon \geq \frac{1-p}{\Pi p} \mu_\delta$, shock impact
 - ▶ $\mu_\varepsilon \gg \mu_\delta$, shock impact
 - ▶ $\Pi p \geq 1-p$, if $\mu_\varepsilon \approx \mu_\delta$, persistence



GMA($\infty,1$) Rule

- ▶ $P_t \geq \left(\prod_{j=0}^{n-1} P_{t-j} \right)^{\frac{1}{n}}$
- ▶ $\ln P_t \geq \frac{1}{n} \sum_{j=0}^{n-1} \ln P_{t-j}$
- ▶ $\ln P_t \geq \mu_1$



GMA($\infty, 1$) Expected Returns

- ▶ $\Phi_{RR_T}(s) =$
 $(1 - \Pi)q[\Phi_\delta(s) + \Phi_\delta(-s)] +$
 $[1 - p(1 - \Pi)][\Phi_\varepsilon(s) + \Phi_\varepsilon(-s)]$
- ▶ $E(RR_T) = -[1 - p(1 - \Pi)][\mu_\varepsilon + \mu_\delta]$



MA Using the Whole History

- ▶ An investor will always expect to lose money using $GMA(\infty,1)$!
- ▶ An investor loses the least amount of money when the return process is a random walk.



Optimal MA Parameters

- ▶ So, what are the optimal n and m ?



A Mathematical Analysis of Linear Technical Indicators

Linear Technical Indicators

- ▶ As we shall see, a number of linear technical indicators, including the Moving Average Crossover, are really the “same” *generalized* indicator using different parameters.



The Generalized Linear Trading Rule

- ▶ A linear predictor of weighted lagged returns
 - ▶ $F_t = \delta + \sum_{j=0}^t d_j X_{t-j}$
- ▶ The trading rule
 - ▶ Long: $B_t = 1$, iff, $F_t > 0$
 - ▶ Short: $B_t = -1$, iff, $F_t < 0$
- ▶ (Unrealized) rule returns
 - ▶ $R_t = B_{t-1} X_t$
 - ▶ $R_t = -X_t$ if $B_{t-1} = -1$
 - ▶ $R_t = +X_t$ if $B_{t-1} = +1$



Buy And Hold

▶ $B_t = 1$



Predictor Properties

- ▶ Linear
- ▶ Autoregressive
- ▶ Gaussian, assuming X_t is Gaussian
- ▶ If the underlying returns process is linear, F_t yields the best forecasts in the mean squared error sense.



Returns Variance

- ▶ $\text{Var}(R_t) = E(R_t^2) - (E(R_t))^2$
- ▶ $= E(B_{t-1}^2 X_t^2) - (E(R_t))^2$
- ▶ $= E(X_t^2) - (E(R_t))^2$
- ▶ $= \sigma^2 + \mu^2 - (E(R_t))^2$



Maximization Objective

- ▶ Variance of returns is inversely proportional to expected returns.
- ▶ The more profitable the trading rule is, the less risky this will be if risk is measured by volatility of the portfolio.
- ▶ Maximizing returns will also maximize returns per unit of risk.



Expected Returns

- ▶ $E(R_t) = E(B_{t-1}X_t)$
- ▶ $= E(B_{t-1}(\mu + \sigma N))$
- ▶ $= \sigma E(B_{t-1}N) + \mu E(B_{t-1})$
- ▶ $E(B_{t-1}) = 1 \times P(F_{t-1} > 0) + -1 \times P(F_{t-1} < 0)$
- ▶ $= P(F_{t-1} > 0) - P(F_{t-1} < 0)$
- ▶ $= 1 - 2 \times P(F_{t-1} < 0)$
- ▶ $= 1 - 2 \times \Phi\left(-\frac{\mu_F}{\sigma_F}\right)$



Truncated Bivariate Moments

- ▶ Johnston and Kotz, 1972, p.116
- ▶ $E(B_{t-1}N) = \iint_{F_t > 0} N - \iint_{F_t < 0} N$
- ▶ $= \sqrt{\frac{2}{\pi}} \rho e^{-\frac{\mu_F^2}{2\sigma_F^2}}$
- ▶ Correlation:
 - ▶ $\rho = \text{Corr}(X_t, F_{t-1})$



Expected Returns As a Weighted Sum

- ▶ $E(R_t) = \sigma E(B_{t-1}N) + \mu E(B_{t-1})$
- ▶ $= \sigma \sqrt{\frac{2}{\pi}} \rho e^{-\frac{\mu_F^2}{2\sigma_F^2}} + \mu \left(1 - 2 \times \Phi \left(-\frac{\mu_F}{\sigma_F} \right) \right)$

a term for volatility

a term for drift



Praetz model, 1976

- ▶ Returns as a random walk with drift.
- ▶ $E(R_t) = \mu(1 - 2f)$, f the frequency of short positions
- ▶ $\text{Var}(R_t) = \sigma^2$



Comparison with Praetz model

▶ Random walk implies $\rho = \text{Corr}(X_t, F_{t-1}) = 0$.

▶ $E(R_t) = \mu \left(1 - 2 \times \Phi \left(-\frac{\mu_F}{\sigma_F} \right) \right)$ the probability of being short

▶ $\text{Var}(R_t) = \sigma^2 + \mu^2 - \left\{ \mu \left(1 - 2 \times \Phi \left(-\frac{\mu_F}{\sigma_F} \right) \right) \right\}^2$

▶ $= \sigma^2 + 4\mu^2 \Phi \left(-\frac{\mu_F}{\sigma_F} \right) \left(1 - \Phi \left(-\frac{\mu_F}{\sigma_F} \right) \right)$

increased variance



Biased Forecast

- ▶ A biased (Gaussian) forecast may be suboptimal.
- ▶ Assume underlying mean $\mu = 0$.
- ▶ Assume forecast mean $\mu_F \neq 0$.

- ▶
$$E(R_t) = \sigma \sqrt{\frac{2}{\pi}} \rho e^{-\frac{\mu_F^2}{2\sigma_F^2}} \leq \sigma \sqrt{\frac{2}{\pi}} \rho$$



Maximizing Returns

- ▶ Maximizing the correlation between forecast and one-ahead return.
- ▶ First order condition:
 - ▶ $\frac{\mu_F}{\sigma_F} = \frac{\mu}{\sigma\rho}$



First Order Condition

▶ Let $x = \frac{\mu_F}{\sigma_F}$

▶ $E(R_t) = \sigma \sqrt{\frac{2}{\pi}} \rho e^{-\frac{x^2}{2}} + \mu(1 - 2 \times \Phi(-x))$

▶ $\frac{d E(R_t)}{dx} = 0$

▶ ~~$\sigma \sqrt{\frac{2}{\pi}} \rho(-x) e^{-\frac{x^2}{2}} + \mu \sqrt{\frac{2}{\pi}} e^{-\frac{x^2}{2}} = 0$~~

▶ $x = \frac{\mu_F}{\sigma_F} = \frac{\mu}{\sigma \rho}$



Fitting vs. Prediction

- ▶ If X_t process is Gaussian, no linear trading rule obtained from a finite history of X_t can generate expected returns over and above F_t .
- ▶ Minimizing mean squared error \neq maximizing P&L.
- ▶ In general, the relationship between MSE and P&L is highly non-linear (Acar 1993).



Technical Analysis

- ▶ Use a finite set of historical prices.
- ▶ Aim to maximize profit rather than to minimize mean squared error.
- ▶ Claim to be able to capture complex non-linearity.
- ▶ Certain rules are ill-defined.



Technical Linear Indicators

- ▶ For any technical indicator that generates signals from a finite linear combination of past prices
 - ▶ Sell: $B_t = -1$ iff $\sum_{j=0}^{m-1} a_j P_{t-j} < 0$
- ▶ There exists an (**almost**) equivalent AR rule.
 - ▶ Sell: $\widetilde{B}_t = -1$ iff $\delta + \sum_{j=0}^{m-2} d_j X_{t-j} < 0$
 - ▶ $X_t = \ln \frac{P_t}{P_{t-1}}$
 - ▶ $\delta = \sum_{j=0}^{m-1} a_j$, $d_j = -\sum_{i=j}^{m-2} a_i$



Conversion Assumption

- ▶ $1 - \frac{P_{t-j}}{P_t} \approx \ln \frac{P_t}{P_{t-j}}$
- ▶ Monte Carlo simulation:
 - ▶ 97% accurate
 - ▶ 3% error.



Example Linear Technical Indicators

- ▶ Simple order
- ▶ Simple MA
- ▶ Weighted MA
- ▶ Exponential MA
- ▶ Momentum
- ▶ Double orders
- ▶ Double MA

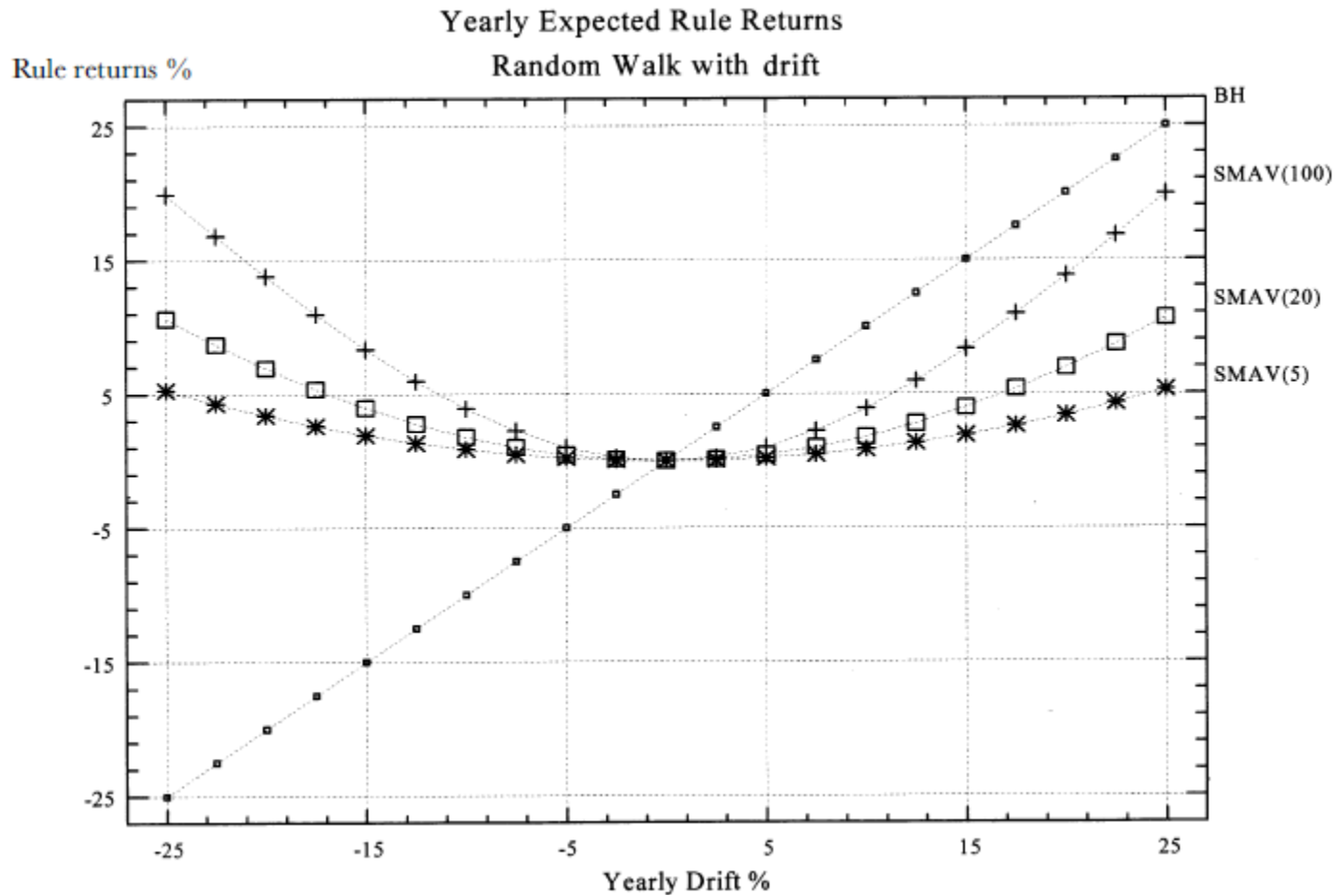


Returns: Random Walk With Drift

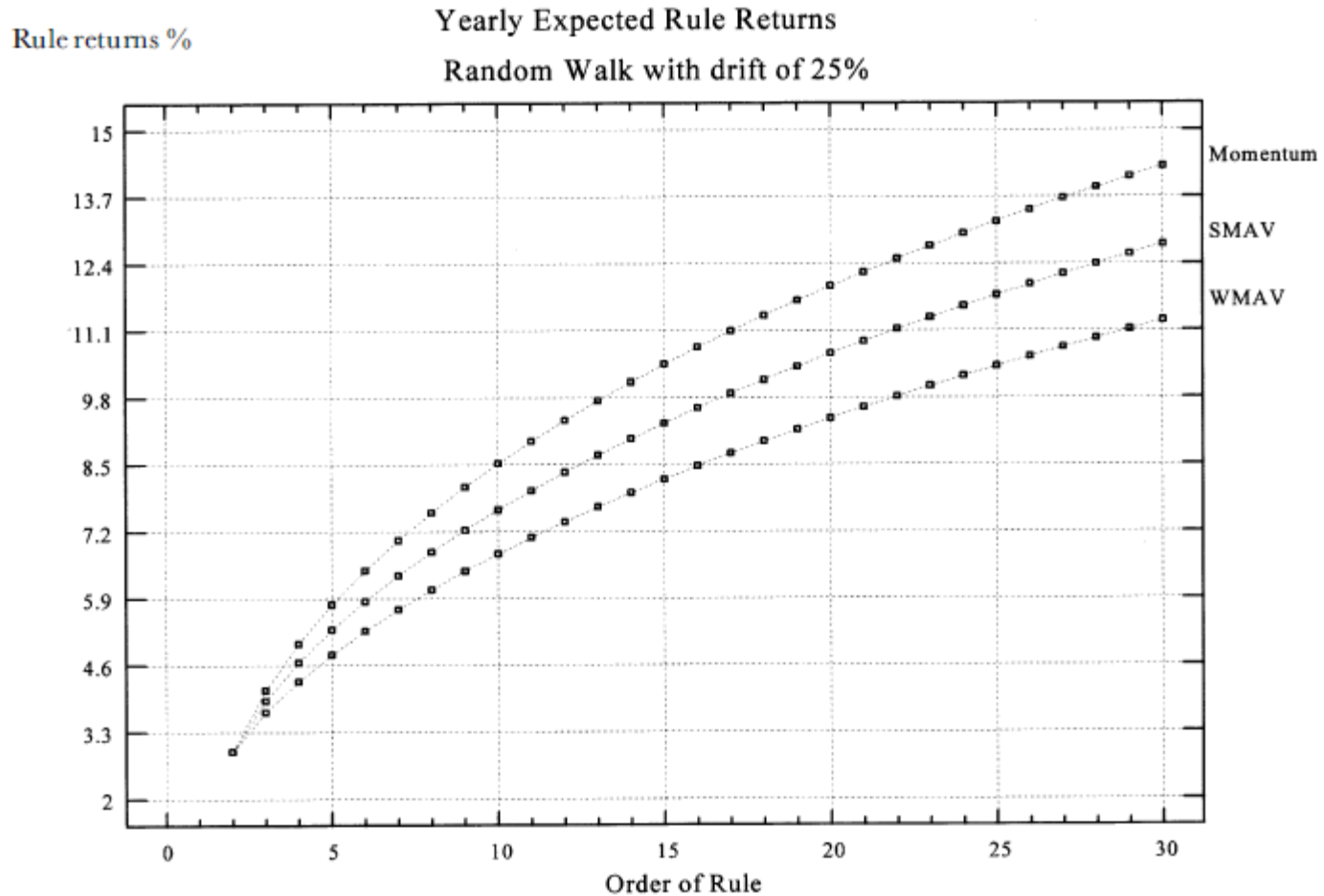
- ▶ $X_t = \mu + \varepsilon_t$
 - ▶ The bigger the order, the better.
 - ▶ Momentum > SMAV > WMAV
- ▶ How to estimate the *future* drift?
 - ▶ Crystal ball?
 - ▶ Delphic oracle?



Results



Results

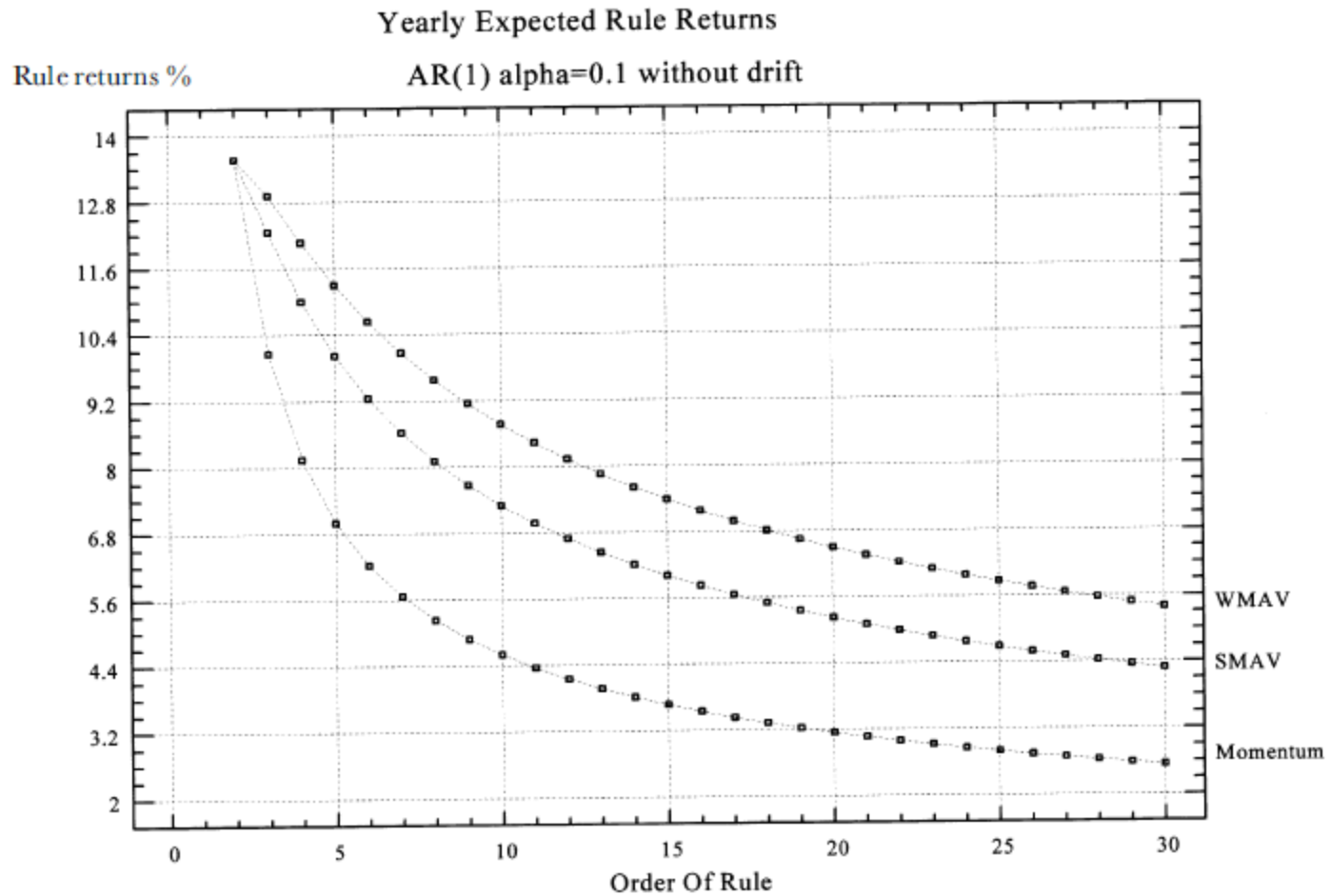


Returns: AR(1)

- ▶ $X_t = \alpha X_{t-1} + \varepsilon_t$
 - ▶ Auto-correlation is required to be profitable.
 - ▶ The smaller the order, the better. (quicker response)



Results



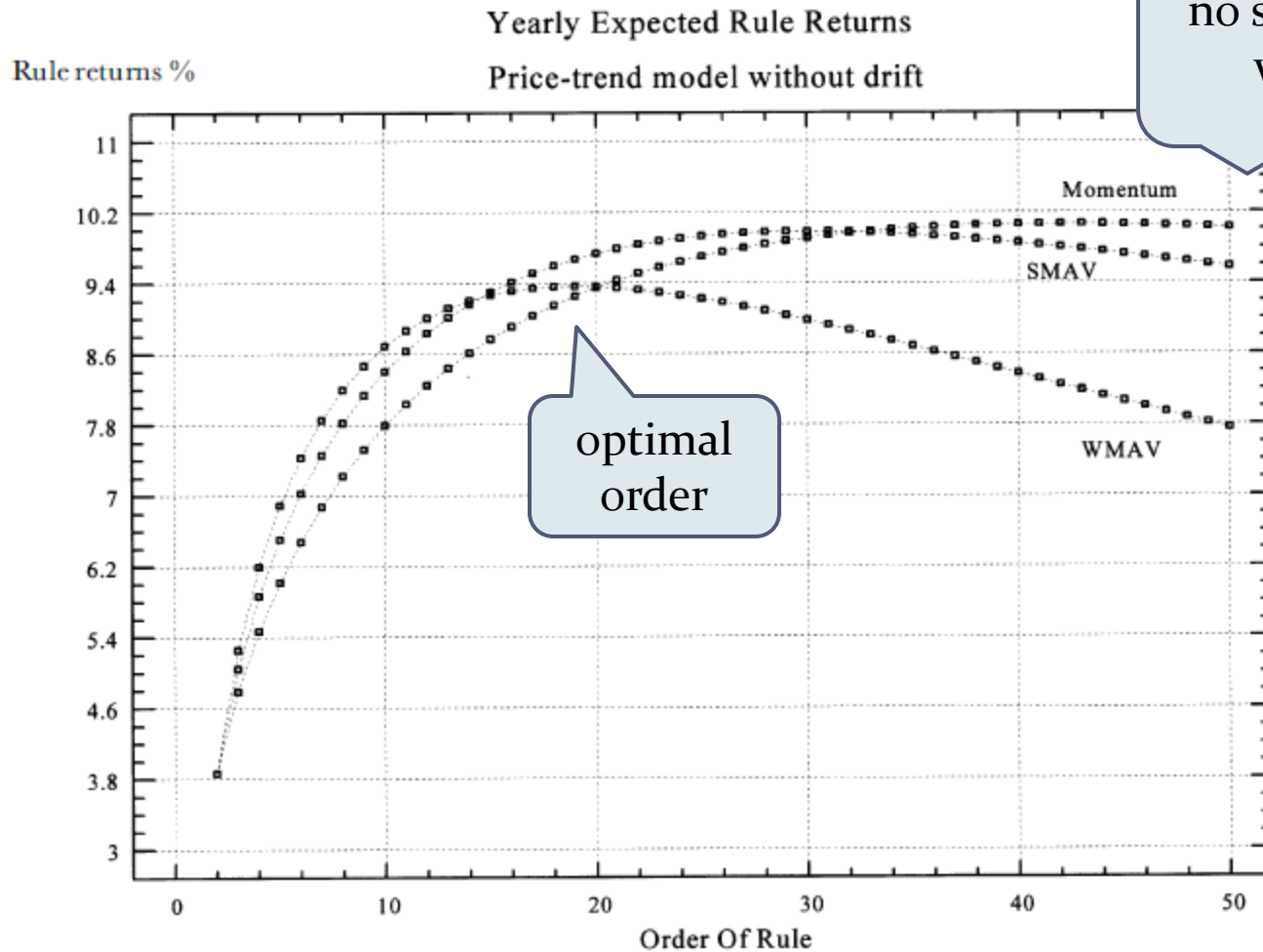
Returns: AR(1, 1)

AR

MA

- ▶ $(X_t - \mu) - p(X_{t-1} - \mu) = \varepsilon_t - q\varepsilon_{t-1}$
- ▶ Prices tend to move in one direction (trend) for a period of time and then change in a random and unpredictable fashion.
 - ▶ Mean duration of trends: $m_d = \frac{1}{(1-p)}$
- ▶ Information has impacts on the returns in different days (lags).
 - ▶ Returns correlation: $\rho_h = Ap^h$

Results



no systematic winner

optimal order

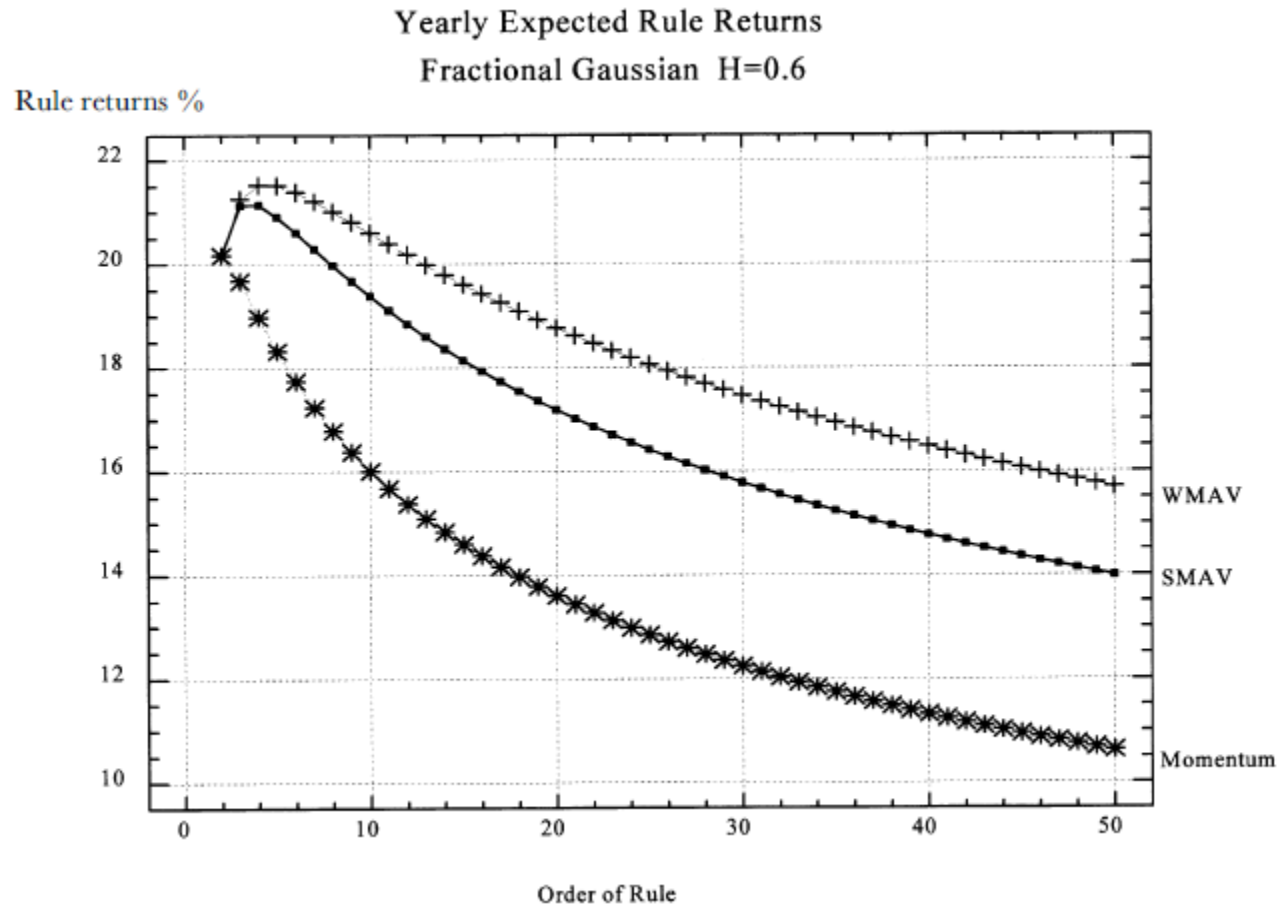


Returns: ARIMA(o, d, o)

- ▶ $\nabla^d(X_t - \mu) = e_t$
- ▶ Irregular, erratic, aperiodic cycles.



Results



Returns: ARCH(p)

$$\blacktriangleright X_t = \mu + \left\{ \sqrt{\alpha_0 + \sum_{i=1}^p \alpha_i (X_{t-i} - \mu)^2} \right\} \varepsilon_t$$

$\blacktriangleright X_t - \mu$ are the residuals

\blacktriangleright When $\mu = 0$, $E(R_t) = 0$.

residual coefficients as a
function of lagged squared
residuals



Returns: AR(2) + GARCH(1,1)

AR(2)

GARCH(1,1)

▶ $X_t = a + b_1 X_{t-1} + b_2 X_{t-2} + \varepsilon_t$

▶ $\varepsilon_t = \sqrt{h_t} z_t$ innovations

▶ $h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1}$

ARCH(1): lagged squared residuals

lagged variance



Results

- ▶ The presence of conditional heteroskedasticity will not drastically affect returns generated by linear rules.
- ▶ The presence of conditional heteroskedasticity, if unrelated to serial dependencies, may be neither a source of profits nor losses for linear rules.



Playing Trend Following Strategies

- ▶ Trend following model requires positive (negative) autocorrelation to be profitable.
 - ▶ What do you do when there is zero autocorrelation?
- ▶ Trend following models are profitable when there are drifts.
 - ▶ How to estimate drifts?
- ▶ It seems quicker response rules tend to work better.
- ▶ Weights should be given to the more recent data.





Conclusions

The Essential Skills

- ▶ Financial intuitions, market understanding, creativity.
- ▶ Mathematics.
- ▶ Computer programming.

An Emerging Field

- ▶ It is a financial industry where mathematics and computer science meet.
- ▶ It is an arms race to build
 - ▶ more reliable and faster execution platforms (computer sciences);
 - ▶ more comprehensive and accurate prediction models (mathematics).
- ▶ Structured products seem to be an evening industry after the financial crisis. Could quantitative trading be another gold mine?

AlgoQuant Demo
